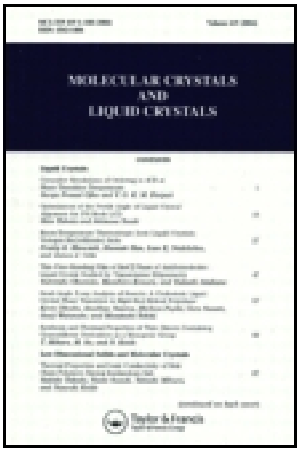

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GUIDED ELECTROMAGNETIC WAVES OF HIGHER ORDERS IN CHOLESTERIC LIQUID CRYSTALS

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Abstract Guided electromagnetic waves (GEW) of optical frequency range in cholesteric liquid crystals (CLC) of finite thickness in conditions of total internal reflection on both boundaries are theoretically examined. Two kinds of interface are discussed : CLC - metal, CLC - dielectric. It's shown that in such films two types of waves can propagate: damped and undamped modes. The main characteristics of GEW of the both types have been investigated in detail, the domains of the existence have been determined. The comparative analysis of GEW and surface waves of semi-infinite samples has been done.

INTRODUCTION

Electromagnetic surface waves at an interface between homogeneous and periodic media have been investigated in several papers[1-6]. Their appearance is determined by two effects: total internal reflection at the boundary and diffraction reflection at the periodic medium (in our case at CLC).

The depth of penetration of the surface wave into CLC may be large, and the influence of the second boundary (if it present) became essential. It is reason the dispersion law is changing compared with the case of surface waves in semi-infinite samples. Besides, it can be expected that eigen modes exist in the structures of finite thickness. The investigation of the properties of such waves is also practically interesting. In the present report GEW of the second order diffraction reflection, arising in CLC film of finite thickness on condition of total internal reflection on both boundaries are examined. The dispersion laws for the most important types of the boundaries are proposed : CLC - metal, CLC - dielectric. The investigation of the problem is carried out for planar cholesteric texture within the framework of the two-wave dynamic diffraction theory of CLC optics.

MAIN EQUATIONS

Examine GEW in a planar CLC film of thickness h surrounded by isotropic homogeneous media with dielectric constant ε_1 and ε_2 . The dielectric tensor of CLC film may be presented in the form:

$$\hat{\varepsilon} = \begin{bmatrix} \bar{\varepsilon} + \bar{\varepsilon}\delta\cos(\tau z-2\varphi) & \bar{\varepsilon}\delta\sin(\tau z-2\varphi) & 0 \\ \bar{\varepsilon}\delta\sin(\tau z-2\varphi) & \bar{\varepsilon} - \bar{\varepsilon}\delta\cos(\tau z-2\varphi) & 0 \\ 0 & 0 & \varepsilon_{\perp} \end{bmatrix}, \quad (1)$$

where τ - the CLC reciprocal lattice vector of the dielectric tensor CLC, δ - dielectric anisotropy, φ - the angle between x-axis and the director at the CLC surface z=0.

It is assumed that the magnetic permeability of CLC is equal to unity. It is assumed also that the total internal reflection takes place on the both boundaries. Eigen modes of second order of diffraction scattering of light in CLC are linearly polarized[5] . The are three polarization cases of Bloch waves' components : 1. $\sigma - \sigma$, 2. $\pi - \pi$, 3. $\sigma - \pi$ ($\pi - \sigma$) (where σ and π are the conventional notations for linear polarizations). Cases 1 and 2 are analogous to TE and TH waves, arising in periodical waveguides with scalar dielectric penetration [7]. Case 3 differs from these two cases, that's why we are examine GEW of (σ - p) - polarization in detail (then we are call these modes as a mixed polarization modes), giving here only some more important results for case of σ and π polarizations.

We examine monochromatic waves, frequency, propagating along the x-axis.

For the case of the mixed polarization field $\mathbf{E}$ should be written in the form of superposition of four eigen solutions:

$$\mathbf{E} = \{\sum_{=1}^{2} C_j (e^{i(\varphi+\frac{\beta}{2})} e^{i(t_j-\tau)z} e_{oj} + e^{-i(\varphi+\frac{\beta}{2})} e^{i(t_j+\tau)z} e_{zj}) + \sum_{j=3}^{4} C_j (e^{i(\varphi-\frac{\beta}{2})} e^{i(t_j-\tau)z} e_{oj} + e^{-i(\varphi-\frac{\beta}{2})} e^{i(t_j+\tau)z} e_{zj})\} e^{i(q_B+\Delta q)x} \quad (2)$$

where Δq and t_j (diffraction corrections to the wave vectors) satisfy the following relations:

$$t_1 = -t_2^* = -\frac{q_B^2}{2\tau}\left(\frac{\delta}{2} - i\frac{\delta^2}{4}\frac{\sin\beta}{\sin\vartheta_B}\right) = -a+i\gamma,$$

$$\Delta q = -\frac{\delta^2}{8} q_B \left(\frac{q_B^2}{4\tau^2} + \frac{\delta^2}{4}\frac{\cos\beta}{\sin\vartheta_B}\right) \quad (3)$$

$$t_3 = -t_1, \quad t_4 = -t_2, \qquad \sin^2\vartheta_B = \tau^2 c^2/\bar{\varepsilon}\omega^2$$

Vectors e_{oj} and e_{zj} are σ - or π - polarized unit vectors and e_{oj} e_{zj}

The expression for the fields in the homogeneous media take the following form:

$$\mathbf{E} = [B_j\hat{y} + A_j (\hat{x} + i\frac{q}{\gamma_j}\hat{z})] e^{i(q_B+\Delta q)x - \gamma_j z}; \quad (4)$$

$$\gamma_j = (-1)^j \sqrt{(q_B + \Delta q)^2 - \bar{\varepsilon}_j \frac{\omega^2}{c^2}}$$

where the index j=1,2 corresponds to the quantities in the homogeneous media above and under the film, and $\hat{x}$, $\hat{y}$, $\hat{z}$ are the unit vectors along corresponding axis. We obtain the dispersion equation for GEW from the continuity conditions on tangential components of electric and magnetic fields at the both boundaries. It looks like

$$\begin{vmatrix} A_{11} & A^*_{11} & A_{12} & A^*_{12} \\ B_{11} & -B^*_{11} & B_{12} & -B^*_{12} \\ A_{21}G_1 & (A_{21}G_1)^* & A_{22}G^*_3 & A^*_{22}\ G_3 \\ B_{21}G^*_2 & -B^*_{21}G_2 & B_{22}G^*_4 & -B^*_{22}\ G_4 \end{vmatrix} = 0 \qquad (5)$$

where

$$A_{jk} = (1+i\nu_j)\, e^{-i\xi_k}, \quad B_{jk} = (i\frac{\gamma_j}{\tau} - 1)\, e^{i\xi_k}$$

$$G_j = e^{-i(\tau+t_j)h}, \quad \nu_j = \frac{\varepsilon_j\ \omega \sin\vartheta_B}{\sqrt{\varepsilon}\ c\ \gamma_j} \qquad (6)$$

$$\xi_1 = \varphi + \frac{\beta}{2}, \quad \xi_2 = \varphi - \frac{\beta}{2}$$

For the case σ- and π- polarizations these equations are extremely simple, but nevertheless they can't be solved analytically.

For case of σ -polarization dispersion equation is

$$[\gamma_1 \cos\frac{2\varphi+\beta}{2} + \tau\sin\frac{2\varphi+\beta}{2}][\gamma_2\cos\frac{2\varphi-\beta+2\tau h}{2} - \tau\sin\frac{2\varphi-\beta+2\tau h}{2}]e^{-\gamma h} -$$

$$[\gamma_1 \cos\frac{2\varphi-\beta}{2} + \tau\sin\frac{2\varphi-\beta}{2}][\gamma_2\cos\frac{2\varphi+\beta+2\tau h}{2} - \tau\sin\frac{2\varphi+\beta+2\tau h}{2}]e^{\gamma h} = 0 \qquad (7)$$

For this case diffraction corrections Δq and γ to the wave vectors satisfy the following relations:

$$2q_B \Delta q = \frac{\delta^2}{4}\bar{\varepsilon}\,\frac{\omega^2}{c^2}\,\mathrm{ctg}^2\vartheta_B \cos\beta$$
$$2\tau\gamma = \frac{\delta^2}{4}\bar{\varepsilon}\,\frac{\omega^2}{c^2}\,\mathrm{ctg}^2\vartheta_B \sin\beta\,, \quad -\pi < \beta < 0 \tag{8}$$

To obtain dispersion equation for π-polarization it is necessary to change γ_m to $\varepsilon/\varepsilon_m$. The Δq and γ may be obtained from formula (8) by multiplying right parts on $\sin^2\vartheta_B$.

For cases of σ - and π- polarization the SGEW field may be looked for in form (2) where the summation is carried out two eigen optical modes (j =1,2) with e_{oj}, $e_{zj} = \sigma, \pi$ and $e_{oj} \parallel e_{zj}$.

To obtain the solution of our problem it is necessary to find parameter β from dispersion equations (5,7,8). This parameter defines diffraction corrections to the wave vectors and the magnitude of electric fields and it depends on the frequency, director orientation at the boundary z=0 and the film thickness.

GEW MAIN PROPERTIES

As it has been mentioned above for all cases of polarization two types of GEW can exist in CLC film : damped modes analogous to surface waves in semi-infinite media and undamped waves with slowly changing magnitude according to the harmonic law.

The damped waves are described by a real parameter β and the undamped modes - by imaginary. There are only two damped modes and infinite numbers of undamped modes for each kinds of polarization.

Each of the damped modes can propagate in limited angular sectors i.e. the forbidden directions of propagation exist for such modes. However one of them can propagate in all directions. The permitted directions of propagation of each of two damped modes are shown at figure 1.

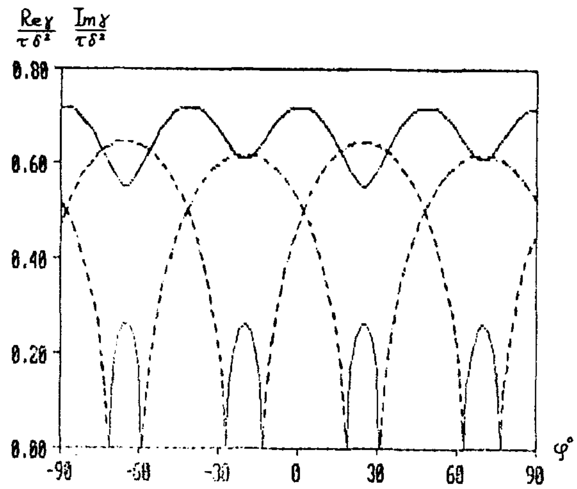

FIGURE 1 Diffraction correction to z-component of wave vectors vs the director's orientation at the surface z = 0.
—— the damped modes ; ---- the undamped modes.
$\bar{\varepsilon}=1.5$, $\varepsilon_1=1.0$, $\delta=0.05$, $h=400\pi^{-1}$, $\omega/c\tau=1.6$.

The dependencies of the diffraction corrections to wave vectors on the angle between the direction of wave propagation and the orientation of the director at the surface z = 0 are shown at figures 1,2. The undamped waves exist in the regions forbidden for the damped modes (fig. 2). It's typical for all cases of polarization. Points, where types of GWE change, connected by the equations:

$$\cos(4\varphi+2\tau h) = -\left[(bh)^2(R^2+Q^2)\right]^{-1}\cdot$$
$$\{[(R^2+Q^2)\cos(2\tau h)+RQ\sin(2\tau h)]\cdot$$
$$[(bh)^2-1] \mp 2bh[(R^2-Q^2)\sin(2\tau h)-RQ\cos(2\tau h)]+$$
$$[(M^2-N^2)\cos(2ah)+2MN\sin(2ah)]\}, \quad (9)$$

where

$$R=(\bar{\varepsilon}+\varepsilon_1)/\bar{\varepsilon}\ ,\ M=(\bar{\varepsilon}\ \varepsilon_1)/\bar{\varepsilon}\ ,\ Q=\gamma_1/\tau+\nu_1,\ b=\gamma/\sin\beta$$

It should be mentioned that the zones of damped and undamped modes change their forms with the change of the frequency, and in the case of mixed polarization their number may be varied. A variation of the thickness by one period of dielectric properties gives the same effects.

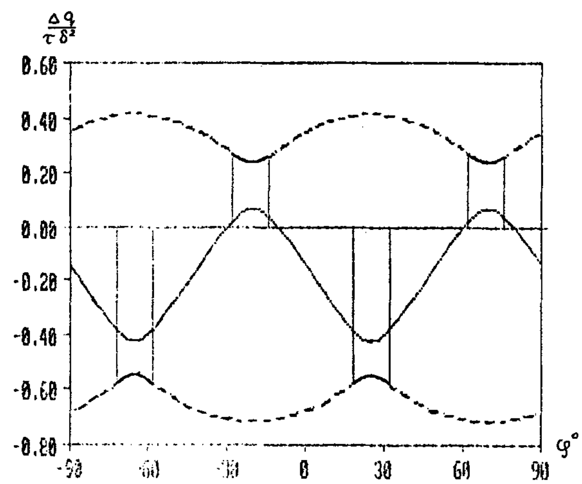

FIGURE 2 Diffraction correction to x-component of wave vectors vs the director's orientation at the surface z = 0. ——— the damped modes ; ----- the undamped modes.
$\bar{\varepsilon}=1.5$, $\varepsilon_1=1.0$, $\delta=0.05$, $h=400\pi^{-1}$, $\omega/c\tau=1.6$.

Let's stop and speak about the distribution of the field with the film thickness for the waves of all types of polarization. In all three cases this distribution is a modulated sinusoid of a space frequency . Its envelope depends on the type of the eigen mode . For σ- and π- polarizations the envelope will be either a damped exponent for surface waves, or a law-frequency sinusoid for undamped waves. For a mixed polarization the envelope is a law-frequency sinusoid with a constant or exponential decreasing amplitude. All these peculiarities are shown on the figures 3 (for damped modes) and 4 (for undamped modes) .

The case when the film boundaries are metallized can be touched upon. In this case the dispersion equations are simpler, but nevertheless they demand also a numerical solution. Main properties of eigen modes of metallized film are the same as its for dielectric film. All results for the metallized film will be published in a separate paper.

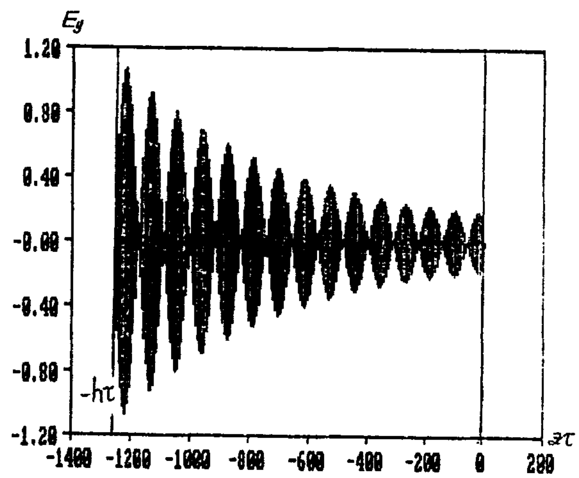

FIGURE 3 The field distribution inside the CLC film for the damped modes of mixed polarization.
$\bar{\varepsilon}=1.5$, $\varepsilon_1=1.0$, $\delta=0.05$, $h=400\pi^{-1}$, $\omega/c\tau=1.6$.

CONCLUSION

The analytical and numeral investigations of GEW of the second diffraction order in cholesteric film reveal an existence of two types modes: damped and undamped. Damped modes analogous to the surface modes in semi-infinite media. Undamped modes never exist in semi-infinite media, they are eigen modes of the film. There are only two damped modes and infinite number undamped modes. Allowed propagation direction of damped modes correlate with an existence domain of the surface modes on the both boundaries of the film. As for the case of the semi-infinite media there are forbidden directions for propagation damped GEW. Two undamped modes exist in

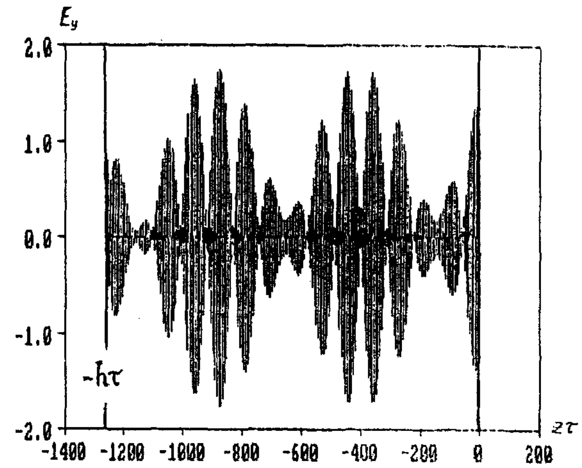

FIGURE 4 The field distribution inside the CLC film for the undamped modes of mixed polarization. $\bar{\varepsilon}$=1.5, ε_1=1.0, δ =0.05, h=400π -1, $\omega/c\tau$ =1.6.

forbidden for damped modes zones. Points, where type of GEW change , lay on the 'surface of change type', its equation depends on propagation direction, frequency and thickness of the film.

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